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STEADY-STATE VERTICAL THERMOELASTIC OSCILLATIONS IN A SYSTEM OF TWO PLANE-PARALLEL LAYERS WITH FRICTION HEAT GENERATION

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We present a mathematical statement and construct a solution for the problem of thermoelasticity in contact interaction between two plane-parallel layers under the action of external loading periodically varying with time.

The problem of static thermoelasticity in contact interaction between two plane-parallel layers uniformly pressed normally against the contact plane with allowance for heat generation and abrasive wear was investigated in [1, 2].

Below we will consider a dynamic contact problem of thermoelasticity for determining steady-state vertical thermoelastic oscillations and temperature fields in a system of two plane-parallel layers exposed to a harmonic normal load (Fig. 1). In this case we take into account heat generation in the contact plane of the layers under the action of friction forces obeying Amonton's law.

Suppose we have a packet of two plane-parallel layers. The lower plane of the packet is rigidly fastened and a pressing load $q = q_0 + q_1 \exp(-i\nu\tau)$ $(0 < q_1 < q_0)$ is applied to the upper plane. The weight of the layers will be taken into account.

We assume that the upper layer moves over the surface of the lower layer with a constant small velocity v_0 in the direction of the z-axis. It is assumed that heat generation occurs in the contact plane of the layers, the thermal contact of the bodies is nonideal, and that heat exchange between the external planes of the layers and the surrounding medium (whose temperature is taken to be equal to zero) follows the Newton law.

We will determine temperature fields, heat fluxes, thermoelastic displacements and stresses in the two-layer packet.

In the given statement of the problem the temperature t_j, displacement V_j and stress $\sigma_y^{(j)}$ $(j = 1, 2)$ will be functions of the coordinate y and time τ. The problem is reduced to solving a system of equations:

$$\partial_y^2 t_j = k_j^{-1} \partial_\tau t_j , \tag{1}$$

$$\partial_y^2 V_j = \beta_j \partial_y t_j + \rho_j g \eta_j^{-1} + c_{1,j}^{-2} \partial_\tau^2 V_j , \tag{2}$$

$$\sigma_y^{(j)} = \eta_j (\partial_y V_j - \beta_j t_j) , \quad j = 1, 2 , \tag{3}$$

under the following boundary and contact conditions:

$$y = -h_1 : \quad \partial_y t_1 = \gamma_1 t_1 , \quad V_1 = 0 ; \tag{4}$$

$$y = h_2 : \quad \partial_y t_2 = -\gamma_2 t_2 , \quad \sigma_y^{(2)} = -q_0 - q_1 \exp(-i\nu\tau) ; \tag{5}$$

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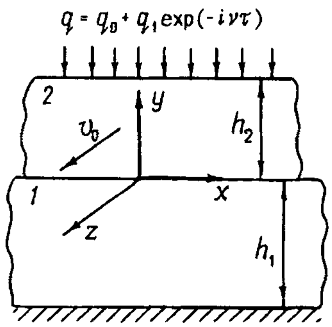


Fig. 1. Diagram of problem of contact interaction between two plane-parallel layers.

$$y = 0: \ \lambda_1 \partial_y t_1 - \lambda_2 \partial_y t_2 = f v_0 (h_2 \rho_2 g + p_0 + p_1 \exp(-i\nu\tau)); \tag{6}$$

$$\lambda_1 \partial_y t_1 + \lambda_2 \partial_y t_2 + h (t_1 - t_2) = 0; \tag{7}$$

$$\sigma_y^{(1)} = \sigma_y^{(2)} = -(h_2 \rho_2 g + p_0 + p_1 \exp(-i\nu\tau)), \quad V_1 = V_2, \tag{8}$$

where

$$\beta_j = \alpha_j (1 + \nu_j)(1 - \nu_j)^{-1}; \ \eta_j = E_j (1 - \nu_j)((1 + \nu_j)(1 - 2\nu_j))^{-1},$$
$$c_{1,j} = \sqrt{\eta_j \rho_j^{-1}}; \ \gamma_j = \overline{\alpha}_j / \lambda_j, \ j = 1, 2. \tag{9}$$

Considering the linearity of the initial equations, boundary and contact conditions, we present the desired solution of the problem in the form of a sum of two terms:

$$t_j(y, \tau) = \overline{t}_j(y) + \overline{\overline{t}}_j(y) \exp(-i\nu\tau),$$
$$V_j(y, \tau) = \overline{V}_j(y) + \overline{\overline{V}}_j(y) \exp(-i\nu\tau), \tag{10}$$
$$\sigma_y^{(j)}(y, \tau) = \overline{\sigma}_y^{(i)}(y) + \overline{\overline{\sigma}}_y^{(i)}(y) \exp(-i\nu\tau), \ j = 1, 2.$$

Substituting relations (10) into Eqs. (1), (2), and (3), satisfying conditions (4)-(8), and equating the expressions at exp $(-i\nu\tau)$, we obtain two boundary-value problems for determining the components of solution (10), which are not given here.

The solution of the first boundary-value problem concerning the contact interaction of layers with allowance for their proper weight and steady-state heat generation is represented by the relations

$$\overline{t}_j = A_j y + B_j, \ \overline{V}_j = 0.5\, y^2 (\beta_j A_j + \rho_j g \eta_j^{-1}) + C_j y + D_j,$$
$$\overline{\sigma}_y^{(j)} = -q_0 - \rho_2 g h_2 + \rho_j g y, \ j = 1, 2; \tag{11}$$

$$A_j = (-1)^{j-1} f v_0 (h_2 \rho_2 g + p_0) a_3^{-1} a_{3-j}, \ B_j = (-1)^{j-1} A_j (h_j + \gamma_j^{-1}),$$

$$a_j = \lambda_j + h (h_j + \gamma_j^{-1}), \ a_3 = \lambda_1 a_2 + \lambda_2 a_1,$$

$$C_j = -\eta_j^{-1}(h_2\rho_2 g + p_0) + \beta_j B_j,\quad D_j = C_1 h_1 - 0.5\, h_1^2(\beta_1 A_1 + \rho_1 g \eta_1^{-1}), \tag{12}$$

in which the component of the contact pressure p_0 is equal to that of the external load q_0.

A simple analysis of Eqs. (10) and (1) shows that the temperature, and, as a result, thermoelastic displacements and stresses of the second boundary-value problem, will be complex functions of a real argument. Introducing the complex parameter $\Delta_j = \sqrt{i\nu k_j^{-1}}$, we obtain the following expressions for determining the temperature of the layers $\bar{\bar{t}}_j(y)$:

$$\bar{\bar{t}}_j(y) = -f v_0 p_1 H_3^{-1} H_{3-j} \frac{\Delta_j \cos(\Delta_j(h_j \pm y)) + \gamma_j \sin(\Delta_j(h_j \pm y))}{\Delta_j \sin(\Delta_j h_j) - \gamma_j \cos(\Delta_j h_j)}, \tag{13}$$

where

$$H_j = \lambda_j \Delta_j - h \frac{\Delta_j \cos(\Delta_j h_j) + \gamma_j \sin(\Delta_j h_j)}{\Delta_j \sin(\Delta_j h_j) - \gamma_j \cos(\Delta_j h_j)};\quad H_3 = \lambda_1 \Delta_1 H_2 + \lambda_2 \Delta_2 H_1;$$

the upper sign in the combination $\pm$ corresponds to the value $j = 1$ and the lower to $j = 2$.

The thermoelastic displacements and stresses take the form:

$$\bar{\bar{V}}_1(y) = -(p_1 c_{1,1}(\nu\eta_1)^{-1} - \nu c_{1,1}^{-1}\varepsilon_1 \bar{\bar{t}}_1(0)) \sin(\nu c_{1,1}^{-1}(h_1 + y)) \cos^{-1}(\nu c_{1,1}^{-1} h_1) +$$
$$+ \varepsilon_1 (d_y \bar{\bar{t}}_1(y) - d_y \bar{\bar{t}}_1(-h_1) \cos(\nu c_{1,1}^{-1} y) \cos^{-1}(\nu c_{1,1}^{-1} h_1)); \tag{14}$$

$$\bar{\bar{V}}_2(y) = -(p_1 c_{1,2}(\nu\eta_2)^{-1} - \nu c_{1,2}^{-1}\varepsilon_2 \bar{\bar{t}}_2(0)) \cos(\nu c_{1,2}^{-1}(h_2 - y)) \sin^{-1}(\nu c_{1,2}^{-1} h_2) +$$
$$+ (q_1 c_{1,2}(\nu\eta_2)^{-1} - \nu c_{1,2}^{-1}\varepsilon_2 \bar{\bar{t}}_2(h_2)) \cos(\nu c_{1,2}^{-1} y) \sin^{-1}(\nu c_{1,2}^{-1} h_2) + \varepsilon_2 d_y \bar{\bar{t}}_2(y);$$

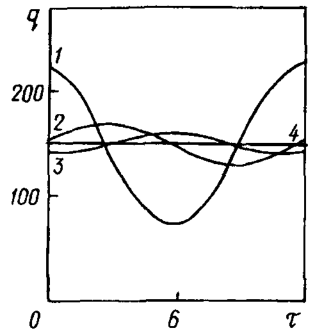
$$\bar{\bar{\sigma}}_y^{(1)}(y) = -p_1 \cos(\nu c_{1,1}^{-1}(h_1 + y)) \cos^{-1}(\nu c_{1,1}^{-1} h_1) +$$
$$+ \eta_1 (\nu c_{1,1}^{-1})^2 \varepsilon_1 (\bar{\bar{t}}_1(0) \cos(\nu c_{1,1}^{-1}(h_1 + y)) \cos^{-1}(\nu c_{1,1}^{-1} h_1) - \bar{\bar{t}}_1(y)) +$$
$$+ \eta_1 \nu c_{1,1}^{-1} \varepsilon_1 d_y \bar{\bar{t}}_1(-h_1) \sin(\nu c_{1,1}^{-1} y) \cos^{-1}(\nu c_{1,1}^{-1} h_1); \tag{15}$$

$$\bar{\bar{\sigma}}_y^{(2)}(y) = -(q_1 \sin(\nu c_{1,2}^{-1} y) + p_1 \sin(\nu c_{1,2}^{-1}(h_2 - y))) \sin^{-1}(\nu c_{1,2}^{-1} h_2) +$$
$$+ \eta_2 (\nu c_{1,2}^{-1})^2 \varepsilon_2 ([\bar{\bar{t}}_2(h_2) \sin(\nu c_{1,2}^{-1} y) + \bar{\bar{t}}_2(0) \sin(\nu c_{1,2}^{-1}(h_2 - y))] \sin^{-1}(\nu c_{1,2}^{-1} h_2) - \bar{\bar{t}}_2(y)),$$

where $\varepsilon_j = \beta_j(\nu^2 c_{1,j}^{-2} - \Delta_j^2)^{-1}$.

The unknown component of the contact pressure p_1 is determined from the condition of the equality of thermoelastic displacements on the contact plane. Using expressions (14), we obtain a formula for defining p_1:

$$p_1 \Big\{\cos(\nu c_{1,1}^{-1} h_1) \cos(\nu c_{1,2}^{-1} h_2) - \eta_2 c_{1,1}(\eta_1 c_{1,2})^{-1} \sin(\nu c_{1,1}^{-1} h_1) \sin(\nu c_{1,2}^{-1} h_2) +$$
$$+ f v_0 \nu c_{1,2}^{-1} \eta_2 [\nu c_{1,1}^{-1} \varepsilon_1 \bar{\bar{t}}_1^*(0) \sin(\nu c_{1,1}^{-1} h_1) \sin(\nu c_{1,2}^{-1} h_2) -$$
$$- \nu c_{1,2}^{-1} \varepsilon_2 \bar{\bar{t}}_2^*(0) \cos(\nu c_{1,1}^{-1} h_1) \cos(\nu c_{1,2}^{-1} h_2) + (\varepsilon_1 d_y \bar{\bar{t}}_1^*(0) -$$



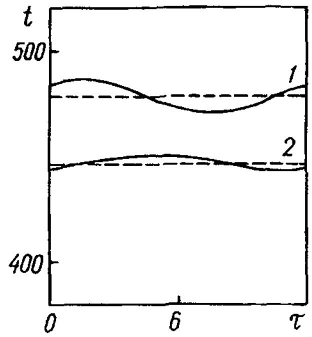


Fig. 2. Time-dependence of heat flux for four values of distance from contact plane ($\nu = \pi/6$ sec^{-1}): 1) $y = 0$, 2) ± 0.01 m, 3) ± 0.02 m, 4) $\pm 0.05-0.1$ m. q, kW/m^2; τ, sec.

Fig. 3. Time-dependence of temperature for two values of distance from contact plane ($\nu = \pi/6$ sec^{-1}): 1) $y = 0$, 2) $y = \pm 0.01$ m (dashed lines correspond to the temperature of the stationary problem). t, °C.

$$- \varepsilon_2 d_y \bar{\bar{t}}_2^* (0)) \cos (\nu c_{1,1}^{-1} h_1) \sin (\nu c_{1,2}^{-1} h_2) - \varepsilon_1 d_y \bar{\bar{t}}_1^* (-h_1) \sin (\nu c_{1,2}^{-1} h_2) +$$

$$+ \nu c_{1,2}^{-1} \varepsilon_2 \bar{\bar{t}}_2^* (h_2) \cos (\nu c_{1,1}^{-1} h_1)]\} = q_1 \cos (\nu c_{1,1}^{-1} h_1) , \tag{16}$$

in which the functions $\bar{\bar{t}}_j^*(y)$ are related to the temperature of the layers $\bar{\bar{t}}_j(y)$ by the relation $\bar{\bar{t}}_j^*(y) = f v_0 p_1 \bar{\bar{t}}_j^*(y)$. Since the parameter ε_j is complex and $\bar{\bar{t}}_j^*(y)$ is a complex function of a real argument, the component of the contact pressure p_1 will also be complex: $p_1 = p_1^{(1)} + i p_1^{(2)}$.

Initial problem (1)-(8) is finally determined from formulas (10), in which the solution of the second boundary-value problem is represented by complex functions of a real argument.

In order to calculate the temperature fields, thermoelastic displacements and stresses, one must use the real parts of expressions (10). This corresponds to a change in the external load according to the law $q = q_0 + q_1 \cos (\nu\tau)$ $(0 < q_1 < q_0)$.

A numerical analysis of the problem is carried out for a steel-steel friction pair ($E_j = 2.06 \cdot 10^5$ MPa, $\nu_j = 0.28$, $\rho_j = 7.8 \cdot 10^3$ kg/m^3, $\lambda_j = 47$ W/(m·K), $k_j = 0.1295 \cdot 10^{-4}$ m^2/sec, $\alpha_j = 11.7 \cdot 10^{-6}$ K^{-1}) and for the values of the main parameters: $q_0 = 0.4$ MPa, $q_1 = 0.2$ MPa, $h = 10$ kW/(m^2·K), $g = 9.81$ m/sec^2, $f = 0.25$, $v_0 = 3$ m/sec, $\gamma_1 = \gamma_2 = 20$ m^{-1}, $h_1 = h_2 = 0.1$ m, $\nu = 0.0001-1000$ sec^{-1}.

A numerical analysis of the solution makes it possible to conclude that the imaginary part $p_1^{(2)}$ of the component of the contact pressure p_1 can be neglected and it may be assumed that $p_1 = p_1^{(1)} = q_1$. In this case the stresses $\bar{\bar{\sigma}}_y^{(j)}$ in the system are invariable over the packet thickness and are equal to the component q_1 of the external load.

The analytical expressions obtained and the analysis of the numerical results show that heat generation as well as the distribution of temperature and heat flux in the two-layer system considered have a fluctuational character. The fluctuations attenuate with distance from the contact plane. Graphs of the variations in time of the heat flux at $\nu = \pi/6$ sec^{-1} are given in Fig. 2, where curve 4 corresponds to the heat flux of stationary problem (11)-(12). A decrease in the frequency ν slows down this attenuation, but when the frequency decreases considerably, oscillations of the heat flux appear, which are independent of the coordinate y and have the frequency of the external load.

The temperature fluctuations lag in phase from changes in the heat flux. Graphs of the temperature distribution are depicted in Fig. 3. As the frequency ν increases, the fluctuation amplitude of the temperature decreases at a fixed coordinate y. A decrease in ν decreases the phase shift between fluctuations of the heat flux and temperature, and increases the temperature amplitude.

Heat generation also causes a phase shift between fluctuations of vertical displacements and stresses. This shift decreases with a decrease in the frequency ν. The amplitude of the change in displacement, being maximum on the loaded surface, decreases with an increase of the parameter ν, and already at $\nu = \pi/6$ sec^{-1} the thermoelastic fluctuations of the displacement can be neglected.

Effects similar to those considered above are observed with a change in the thermal diffusivity parameter. An increase in k_j causes the same effects that are manifested with a decrease in the fluctuation frequency ν. As we know, in nonstationary problems of heat conduction an increase in the thermal diffusivity parameter decreases the time needed for the temperature to attain a stationary value. In the given problem a decrease in the frequency ν increases the residence time of the system in a "stationary" state, which is characterized by relative constancy of the external load. Thermoelastic oscillations of the system can be considered in this case as transition from one thermoelastic stationary state to another, which is characterized by relatively constant loads.

NOTATION

$q = q_0 + q_1 \exp(-i\nu\tau)$, external pressure; $p = p_0 + p_1 \exp(i\nu\tau)$, contact pressure; ν, frequency of fluctuations of the external load; τ, time; t_j, $j = 1, 2$, temperature; V_j, $j = 1, 2$, displacement; $\sigma_y^{(j)}$, $j = 1, 2$, stress; y, spatial coordinate; v_0, velocity; g, free fall acceleration; h, thermal conductivity of the contact plane; f, friction coefficient; h_j, $j = 1, 2$, thickness of the layers; ρ_j, $j = 1, 2$, density; E_j, $j = 1, 2$, Young modulus; ν_j, α_j, $\overline{\alpha}_j$, λ_j, k_j, $j = 1, 2$, coefficients of Poisson, linear temperature expansion, heat transfer, thermal conductivity and of thermal diffusivity; $c_{l,j}$, $j = 1, 2$, speed of propagation of longitudinal waves.

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